AD-A185 562 DEVELOPMENT OF SYMBOLIC COMPUTATION METHODS FOR NONLINEAR DYNAMICS(U) CORNELL UNIV ITHACA NY DEPT OF THEORETICAL AND APPLIED MECHAN... R H RAND 14 JUL 87 1/1

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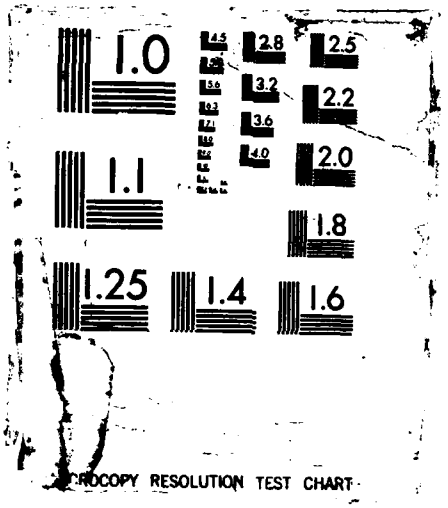
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REPORT DOCUMENTATION PAGE

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2a. SECURITY CLASSIFICATION AU[illegible]	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
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Development of Symbolic Computation Metjods for Nonlinear Dynamics

12. PERSONAL AUTHOR(S)
Prof R H Rand

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17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Under the supervision of principal investigator R.H. Rand, software has been written in MACSYMA which automatically performs normal form computations for systems of nonlinear nonautonomous differential equations. Together with postdoctoral research associate W.L. Keith, we have produced a package which permits the user to perform Taylor expanded near-identity transformations with unevaluated coefficients on a system of autonomous ODE's (valid to terms of arbitary order), and then to choose the transformation coefficients so that the resulting system is in normal form. A summary of this work, including the program listing has been published.

This work has been applied to thenonlinear parametric stiffness control of flexible systems by Professors Moon and Rand, and to the dynamics of coupled van der Pol oscillators, by graduate student T. Chakraborty.

This work is currently being extended to cover a class of nonautonomous periodic systems by graduate student J. Goon. Inthis case the computer algebra code performs both Taylor&Fourier expansion.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
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Cornell University

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Ithaca, NY 14853-2801

607/255-5014

AFOSR-TR- 87-1344

20 May 1987
14 July 1987

Ms. Paulette Bowman
Air Force Office of
Scientific Research
Building 410
Bolling AFB, DC 20332

Dear Ms. Bowman:

RE: AFOSR-84-0311
R.H. Rand

Enclosed you will find a copy of the final technical report for the referenced grant. The report was transmitted to Captain D. R. McGhee on 21 March 1986.

Also enclosed are two copies of DD Form 882 Final Report of Inventions and Subcontracts. The report covers the period 30 July 1984 thru 29 December 1985.

Should you have questions, please feel free to contact me.

Sincerely,

Joseph W. Fitzgerald

Joseph W. Fitzgerald
Grant & Contract Officer

/j

Enclosures

Cornell University

College of Engineering

Department of Theoretical and
Applied Mechanics

Thurston Hall
Ithaca, NY 14853-1503

607/256-5062

21 March 1986

TO: D.R.McGhee, Capt.,USAF
FROM: R.H.Rand
SUBJECT: Final Scientific Report for Grant No. AFOSR-84-0311

The above report is attached.

R. Rand

Final Report
Grant No. AFOSR-84-0311
"Development of Symbolic Computation Methods for Nonlinear Dynamics"
Principal Investigator: R.H. Rand
Dept. Theoretical & Applied Mechanics
Cornell University, Ithaca NY 14853
607-255-7145

This grant was used to purchase the following equipment:
SYMBOLICS 3670 Mainframe Computer (including 6 Mbytes of memory and a 470 Mbyte hard disk)
Laser Graphics Printer
Ethernet board
and the following associated software:
MACSYMA
FORTRAN
PASCAL

The equipment has been used to facilitate computation on the following projects:

1. Normal Forms

Under the supervision of principal investigator R.H. Rand, software has been written in MACSYMA which automatically performs normal form computations for systems of nonlinear, nonautonomous differential equations. Together with postdoctoral research associate W.L. Keith, we have produced a package which permits the user to perform Taylor expanded near-identity transformations with unevaluated coefficients on a system of autonomous ODE's (valid to terms of arbitary order), and then to choose the transformation coefficients so that the resulting system is in normal form. A summary of this work, including the program listing has been published [1].

This work has been applied to the the nonlinear parametric stiffness control of flexible systems by Professors Moon and Rand [2], and to the dynamics of coupled van der Pol oscillators by graduate student T. Chakraborty.

This work is currently being extended to cover a class of nonautonomous periodic systems by graduate student J. Goon. In this case the computer algebra code performs both Taylor and Fourier expansions.

2. Determinacy of Degenerate Equilibria

We have also used MACSYMA on the 3670 to study the problem of the determinacy of degenerate equilibria with linear part

$$x' = y, \quad y' = 0. \tag{1}$$

Takens [3] has shown that any nonlinear autonomous system with such a linear part can be put in the normal form:

$$x' = y + b_2 x^2 + b_3 x^3 + b_4 x^4 + \ldots \quad (2)$$

$$y' = a_2 x^2 + a_3 x^3 + a_4 x^4 + \ldots \quad (3)$$

Since this system is not structurally stable, it is not clear which terms can be truncated without affecting the topological nature of the flow. Takens showed that if $a_2 \neq 0$, then the flow is topologically equivalent to that of the truncated system:

$$x' = y, \quad y' = a_2 x^2. \quad (4)$$

We extended his result in the case that

$$a_2 = a_3 = a_4 = \ldots = a_n = 0, \quad a_{n+1} \neq 0. \quad (5)$$

E.g. we showed that if $a_2 = 0$ but $a_3 \neq 0$, then the flow (2)-(3) is equivalent to the truncated system:

$$x' = y + b_2 x^2, \quad y' = a_3 x^3. \quad (6)$$

The procedure involved transforming from cartesian to polar coordinates in the neighborhood of the singular point and Taylor expanding the polar flow (called blowing up the singularity), then testing for determinacy. If the resulting system is not structurally stable, then we blow it up again. In some cases this required 9 consecutive transformations and Taylor expansions, a computation which could not have been done without a MACSYMA program. This work will soon appear in print [4].

3. Hilbert's 16th Problem

Professors Guckenheimer and Rand have been using MACSYMA to approach Hilbert's 16th problem from a bifurcation point of view. The problem is to find the maximum number of limit cycles exhibited by a system of 2 autonomous ODE's with quadratic right hand sides:

$$x' = a x^2 + b y^2 + c x y + d x + e y + f \quad (7)$$

$$y' = A x^2 + B y^2 + C x y + D x + E y + F \quad (8)$$

This problem has a long history including a paper by Petrovsky and Landis [5] which supposedly showed that the maximum number of limit cycles was 3, but was then claimed to

have errors in it by Moser [6]. The question was illuminated by the recent example of Songling [7], which exhibits 4 limit cycles.

We have been using bifurcation theory to investigate this problem in the following way: We look for the standard type of bifucations which give rise to limit cycles (Hopfs and saddle connections), but require the bifurcations to be as singular as possible. E.g. in the case of the usual Hopf bifurcation, a single condition generates a single limit cycle. However, we have shown that requiring 2 additional conditions will give a bifucation involving the birth of 3 concentric limit cycles. This computation pushes the 3670 to its limits, and certainly could not have done on a smaller machine. Some algebraic expressions which result are 15 Kbytes in length.

4. High Precision Numerical Work on MACSYMA

Professors Parlange and Rand have worked on a problem in scattering theory [8] which involved a slowly convergent series. The leading terms were of order 10^{50} although the series summed to a value of order unity. In order to keep track of the differences between successive terms, we used MACSYMA's ability to work in arbitrary precision. We summed 170 terms, keeping 100 digits for each term. Note that even quadruple precision in FORTRAN would not have been sufficient here.

5. Liapunov-Schmidt Method in Dynamics Problems

Professor Rand and Dr. D. Armbruster have worked on formalizing the Liapunov-Schmidt method to problems involving Hopf and other bifurcations in systems of nonlinear partial differential equations. This method, based on the Fredholm alternative theorem, offers an alternative to traditional perturbation methods such as Linstedt's method (cf. [9]) or the method of averaging.

References

1. Normal Form and Center Manifold Calculations on MACSYMA
   R.H. Rand, W.L. Keith
   in "Applications of Computer Algebra", R. Pavelle, ed., Kluwer Academic Publishers, 309-328 (1985)

2. Parametric Stiffness Control of Flexible Structures
   F.C.Moon, R.H.Rand
   in Proceedings of the Workshop on Identification and Control of Flexible Space Structures, G. Rodriguez, ed., NASA JPL Publication 85-29, Vol.II, pp.329-342, (1985)

3. Singularities of Vector Fields
F.Takens
Publ.Math.Inst.Hautes Etudes Sci. 43:47-100 (1974)

4. Determinacy of Degenerate Equilibria with Linear Part $x'=y$, $y'=0$ Using MACSYMA
R.H. Rand, W.L. Keith
Applied Mathematics and Computation (to appear)

5. On the Number of Limit Cycles of the Equation $dy/dx=P/Q$, where P and Q are Polynomials of the Second Degree
I.G.Petrovsky, E.M.Landis
Amer.Math.Soc.Transl.Ser.2, 10:177-221 (1958)

6. A Survey of Quadratic Systems
W.A.Coppel
J.Diff.Eq. 2:293-304 (1966)

7. A Concrete Example of the Existence of Four Limit Cycles for Plane Quadratic Systems
S.Songling
Scientia Sinica 23:153-158 (1980)

8. Numerical Corrections of Wu's Coefficients for Scattering of High-Frequency Waves from Spheres and Cylinders
I.G.Lisle, J.Y.Parlange, R.H.Rand, M.B.Parlange, W.L.Hogarth, R.D.Braddock, H.P.Gottlieb
Physical Review Letters 55:555-557 (1985)

9. Book: Computer Algebra in Applied Mathematics: An Introduction to MACSYMA
R.H.Rand
Research Notes in Mathematics No.94, 181pp.
Pitman Publishing Inc. (1984)

REPORT OF INVENTIONS AND SUBCONTRACTS

(Pursuant to "Patent Rights" Contract Clause) (See Instructions on Reverse Side.)

FORM APPROVED OMB NO. 0704-0016

1a. NAME OF CONTRACTOR/ SUBCONTRACTOR	c. CONTRACT NUMBER	2a. NAME OF GOVERNMENT PRIME CONTRACTOR	e. CONTRACT NUMBER	3. TYPE OF REPORT (check one)
Cornell Univ.	AFOSR-84-0311	same as 1a.		☐ INTERIM ☒ FINAL
b. ADDRESS (include Zip Code) OSP, 120 Day Hall Ithaca, NY 14853	d. AWARD DATE (YYMMDD) 840730	b. ADDRESS (include Zip Code)	d. AWARD DATE (YYMMDD)	4. REPORTING PERIOD (YYMMDD) FROM 840730 TO 851229

SECTION I - SUBJECT INVENTIONS

5. "SUBJECT INVENTIONS" REQUIRED TO BE REPORTED BY CONTRACTOR/SUBCONTRACTOR *(If "None", so state)*

a. NAME OF INVENTOR(S) (Last, First, M.I.)	b. TITLE OF INVENTION(S)	c. DISCLOSURE NO., PATENT APPLICATION SERIAL NO. OR PATENT NO.	d. ELECTION TO FILE PATENT APPLICATIONS				e. CONFIRMATORY INSTRUMENT OR ASSIGNMENT FORWARDED TO CONTRACTING OFFICER	
			UNITED STATES		FOREIGN			
			YES	NO	YES	NO	YES	NO
NONE								

f. EMPLOYER OF INVENTOR(S) NOT EMPLOYED BY CONTRACTOR/SUBCONTRACTOR.

i. NAME OF INVENTOR (Last, First, M.I.)	ii. NAME OF INVENTOR (Last, First, M.I.)	g. ELECTED FOREIGN COUNTRIES IN WHICH A PATENT APPLICATION WILL BE FILED.	
		i. TITLE OF INVENTION	ii. FOREIGN COUNTRIES OF PATENT APPLICATION
iii. NAME OF EMPLOYER	iii. NAME OF EMPLOYER		
iv. ADDRESS OF EMPLOYER (include Zip Code)	iv. ADDRESS OF EMPLOYER (include Zip Code)		

SECTION II – SUBCONTRACTS *(Containing a "Patent Rights" clause)*

6. SUBCONTRACTS AWARDED BY CONTRACTOR/SUBCONTRACTOR. *(If "None", so state)*

a. NAME OF SUBCONTRACTOR(S)	b. ADDRESS (include Zip Code)	c. SUBCONTRACT NO.(S)	d. DAR "PATENT RIGHTS"		e. DESCRIPTION OF WORK TO BE PERFORMED UNDER SUBCONTRACT(S)	f. SUBCONTRACT DATES (YYMMDD)	
			CLAUSE NO.	DATE (YYMM)		AWARD	ESTIMATED COMPLETION
NONE							

SECTION III – CERTIFICATION

7. CERTIFICATION OF REPORT BY CONTRACTOR/SUBCONTRACTOR. *(Not required if ☐Small Business or ☐Non-Profit organization.) (Check appropriate box)*

a. NAME OF AUTHORIZED CONTRACTOR/SUBCONTRACTOR OFFICIAL (Last, First, M.I.)	c. I certify that the reporting party has procedures for prompt identification and timely disclosure of "Subject Inventions," that such procedures have been followed and that all "Subject Inventions" have been reported.
Feocco, Frank J.	
b. TITLE Associate Director Office of Sponsored Programs	SIGNATURE OF AUTHORIZED CONTRACTOR/SUBCONTRACTOR OFFICIAL — DATE (YYMMDD) 870520

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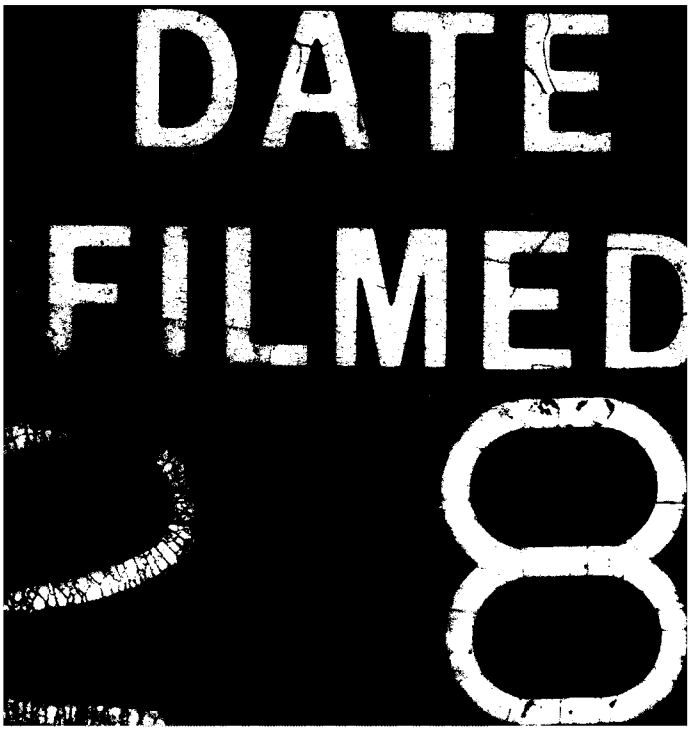
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